BASIC CONCEPTS & CHARGE

$e = 1.6\times10^{-19}$ C (proton: $+e$, electron: $-e$)
Charge quantisation: $q = ne,\ n\in\mathbb{Z}$
Conservation: total charge is constant in isolated system

Properties at a Glance

- Scalar, additive, relativistically invariant
- $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$
- $k = \dfrac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2\text{C}^{-2}$
- Relative permittivity: $\varepsilon_r = \varepsilon/\varepsilon_0 \geq 1$

COULOMB'S LAW

$$F = k\frac{q_1q_2}{r^2} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$$

In medium: $F_{\text{med}} = \dfrac{F_{\text{vac}}}{\varepsilon_r}$

- Central, conservative, inverse-square law
- Valid for point charges / spherically symmetric charges
- Superposition: $\vec F_{\text{net}} = \sum \vec F_i$ (vector sum)

Trick: For 3 charges in equilibrium on a line, middle charge always opposite in sign to the outer two.

ELECTRIC FIELD

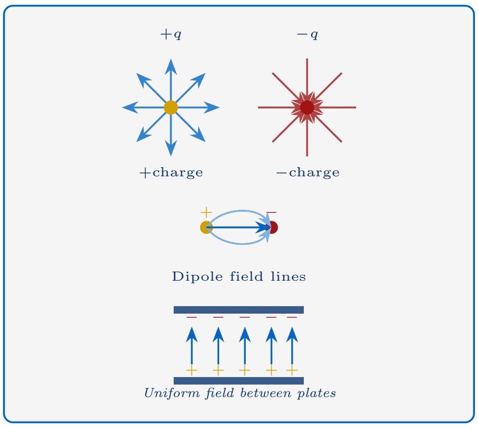
$$\vec E = \frac{\vec F}{q_0} = k\frac{q}{r^2}\hat r \qquad [\text{N/C} = \text{V/m}]$$

Fields from Standard Distributions

Point charge q	$E = \dfrac{kq}{r^2}$
Infinite line (λ)	$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$
Infinite sheet (σ)	$E = \dfrac{\sigma}{2\varepsilon_0}$
Conducting plate (σ)	$E = \dfrac{\sigma}{\varepsilon_0}$ (outside)
Ring (axis, dist. x)	$E = \dfrac{kQx}{(R^2+x^2)^{3/2}}$
Disc (axis, dist. x)	$E = \dfrac{\sigma}{2\varepsilon_0}\left(1 - \dfrac{x}{\sqrt{R^2+}}\right.$
Solid sphere (outside)	$E = \dfrac{kQ}{r^2};\quad r > R$
Solid sphere (inside)	$E = \dfrac{\rho r}{3\varepsilon_0};\quad r < R$
Spherical shell (inside)	$E = 0$

Ring axis max E: at $x = R/\sqrt2$
$$E_{\max} = \frac{kQ}{3\sqrt3\,R^2/2}$$

ELECTRIC FIELD LINE DIAGRAMS



Field Line Properties

- Start at +, end at − (or ∞)
- Never intersect; closer lines ⇒ stronger E
- Perpendicular to equipotential surfaces
- Number of lines ∝ magnitude of charge

ELECTRIC DIPOLE

Dipole moment: $\vec p = q(2a)\hat p$ [C·m]
Direction :-q → +q

Electric Field of Dipole

Axial (end-on): $E = \dfrac{2kp}{r^3}$ (along $\vec p$)

Equatorial (broadside): $E = \dfrac{kp}{r^3}$ (opp. to $\vec p$)

General angle θ:
$$E_r = \frac{2kp\cos\theta}{r^3},\quad E_\theta = \frac{kp\sin\theta}{r^3}$$
$$E = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$$
$$\tan\alpha = \frac{\tan\theta}{2}$$

Key: $E_{\text{axial}} = 2E_{\text{equatorial}}$ (same r)
Both $\propto 1/r^3$ for short dipole ($r \gg a$)

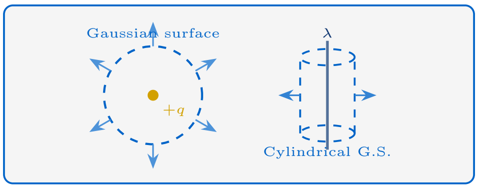
Dipole in External Field

Torque: $\tau = pE\sin\theta,\ \vec\tau = \vec p\times\vec E$
PE: $U = -pE\cos\theta = -\vec p\cdot\vec E$
Stable eq.: $\theta = 0$; Unstable: $\theta = 180$
Force (non-uniform E): $F = p\dfrac{dE}{dx}$

GAUSS'S LAW

$$\oint_S \vec E\cdot d\vec A = \frac{Q_{\text{enc}}}{\varepsilon_0}$$
$$\Phi_E = \frac{Q_{\text{enc}}}{\varepsilon_0} \qquad [\text{Nm}^2/C]$$

- Q_{enc} = only charge *inside* Gaussian surface
- E on surface can be due to all charges
- Best applied when symmetry guarantees E = const on surface



APPLICATIONS OF GAUSS'S LAW

System	Result
Point charge	$E = \dfrac{kQ}{r^2}$
Spherical shell (out)	$E = \dfrac{kQ}{r^2}$
Spherical shell (in)	$E = 0$
Solid sphere (out)	$E = \dfrac{kQ}{r^2}$
Solid sphere (in)	$E = \dfrac{\rho r}{3\varepsilon_0}$
Infinite line λ	$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$
Infinite plane σ	$E = \dfrac{\sigma}{2\varepsilon_0}$
Parallel plates	$E = \dfrac{\sigma}{\varepsilon_0}$ (between)
	$E = 0$ (outside)

ELECTRIC POTENTIAL

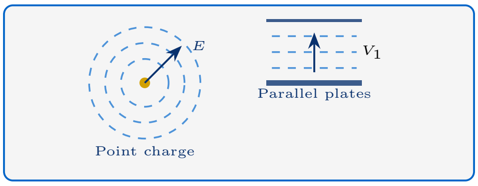
$$V = \frac{W_{\infty\to P}}{q_0} = -\int_\infty^P \vec E\cdot d\vec r$$

Unit: Volt (V) = J/C Scalar quantity

Potential — Standard Distributions

Point charge	$V = \dfrac{kq}{r}$
System of charges	$V = k\sum_i \dfrac{q_i}{r_i}$
Spherical shell (out)	$V = \dfrac{kQ}{r}$
Spherical shell (on/in)	$V = \dfrac{kQ}{R}$ (const)
Solid sphere (out)	$V = \dfrac{kQ}{r}$
Solid sphere (surface)	$V = \dfrac{kQ}{R}$
Solid sphere (inside)	$V = \dfrac{kQ}{2R}\left(3 - \dfrac{r^2}{R^2}\right)$
Ring (axis, dist x)	$V = \dfrac{kQ}{\sqrt{R^2+x^2}}$
Dipole (axial)	$V = \dfrac{kp}{r^2}$
Dipole (equatorial)	$V = 0$
Dipole (general θ)	$V = \dfrac{kp\cos\theta}{r^2}$

EQUIPOTENTIAL SURFACES



- $W = 0$ to move charge along equipotential

- $\vec{E} \perp$ equipotential surface always
- Closer surfaces $\Rightarrow$ stronger E
- Conductor surface is equipotential; E inside $= 0$

RELATION BETWEEN E AND V

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = -\nabla V \quad \text{(gradient relation)}$$

$$V_A - V_B = \int_B^A \vec{E} \cdot d\vec{r}$$

- E directed from high V to low V
- Uniform field: $E = \dfrac{\Delta V}{d}$
- $V =$ const in region $\Rightarrow E = 0$ there

Trick: Slope of V–r graph $= -E$
If V vs x is given, $E = -dV/dx$

ELECTROSTATIC ENERGY & PE

Potential Energy of System

Two charges: $U = \dfrac{kq_1q_2}{r}$

Three charges: $U = k\left(\dfrac{q_1q_2}{r_{12}} + \dfrac{q_2q_3}{r_{23}} + \dfrac{q_1q_3}{r_{13}}\right)$

n charges: $U = \dfrac{1}{2}\sum_{i\neq j} \dfrac{kq_iq_j}{r_{ij}}$

Work & Energy

$W = q_0(V_A - V_B) = q_0\,\Delta V$

KE gained $= q\Delta V$

1 eV $= 1.6 \times 10^{-19}$ J

Energy Density of Electric Field

$$u = \frac{1}{2}\varepsilon_0 E^2 \quad [\text{J/m}^3]$$

$$Total energy : \text{U} = \frac{1}{2}\varepsilon_0 \int E^2\, dV$$

Self-Energy of Spherical Shell

$$U_{\text{self}} = \frac{kQ^2}{2R} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

CONDUCTORS IN ELECTROSTATICS

- $E_{\text{inside}} = 0$; $V_{\text{inside}} = V_{\text{surface}}$
- Charge resides on outer surface only
- E just outside: $\dfrac{\sigma}{\varepsilon_0}$ (normal to surface)
- Sharp points $\Rightarrow$ high σ $\Rightarrow$ high E (corona)
- Cavity with no charge: $E = 0$ (Faraday cage)

MOST COMMON MISTAKES

Watch Out!

1. $E = 0 \not\Rightarrow V = 0$ (inside shell: $E = 0$, $V = kQ/R$)
2. $V = 0 \not\Rightarrow E = 0$ (dipole equatorial: $V = 0$, $E \neq 0$)
3. Gauss's law: Q_{enc} determines flux, not E alone
4. Dipole formulas valid only for $r \gg a$ (short dipole)
5. Work by electric force $= -\Delta U$ (not $+\Delta U$)
6. Potential is **scalar**; add algebraically, not as vectors
7. Inside conductor $E = 0$, not $V = 0$
8. Field lines enter/exit conductor **perpendicularly**

QUICK REVISION TABLE

Quantity	Pt. charge	Dipole
E (axial)	kq/r^2	$2kp/r^3$
E (equat.)	kq/r^2	kp/r^3
V (axial)	kq/r	kp/r^2
V (equat.)	kq/r	0
E falls as	r^{-2}	r^{-3}
V falls as	r^{-1}	r^{-2}

Formula	Units
$F = kq_1q_2/r^2$	N
$E = F/q$	N/C = V/m
$V = W/q$	V = J/C
$\Phi = Q/\varepsilon_0$	Nm^2/C
$u = \frac{1}{2}\varepsilon_0 E^2$	J/m^3
$p = q \cdot 2a$	C·m
$\tau = pE\sin\theta$	N·m
$U = -pE\cos\theta$	J

Constants:
$k = 9 \times 10^9\ \text{Nm}^2\text{C}^{-2}$
$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$
$e = 1.6 \times 10^{-19}$ C 1 eV $= 1.6 \times 10^{-19}$ J